Virtually Free pro-p groups whose Torsion Elements have finite Centralizer

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February 2, 2008

Abstract

A finitely generated virtually free pro-p group with finite centralizers of its torsion elements is the free pro-p product of finite p-groups and a free pro-p factor.

1 Introduction

The objective of this paper is to give a complete description of a finitely generated virtually free pro-p group whose torsion elements have finite centralizers. Our main result is the following

Theorem 1.1 *Let G be a finitely generated virtually free pro-p group such that the centralizer of every torsion element in G is finite. Then G is a free pro-p product of subgroups which are finite or free pro-p.*

This is a rather surprising result from a group theoretic point of view, since the theorem does not hold for abstract groups (as well as for profinite groups): an easy counter example is given in Section 5. However, from a Galois theoretic point of view it is not so surprising. Indeed, the finite centralizer condition for torsion elements arises naturally in the study of absolute Galois groups. In particular, D. Haran [2] (see also I. Efrat in [1] for a different proof) proved the above theorem for the case when G is an extension of a free pro-2 group with a group of order 2.

The proof of Theorem 1 explores a connection between p-adic representations of finite p-groups and virtually free pro-p groups, which gives a new approach to study virtually free pro-p groups. This connection enables us to use the following beautiful result:

*The first author would like to thank for the hospitality at the UNC during February 2003
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Theorem 1.2 *[([9] A. Weiss)] Let G be a finite p-group, N a normal subgroup of G and let M be a finitely generated $\mathbf{Z}_p[G]$-module. Suppose that M is a free N-module and that M^N is a permutation lattice for G/N. Then M is a permutation lattice for G.*

Here M^N means the fixed submodule for N, and a *permutation lattice* for G means a direct sum of G-modules, each of the form $\mathbf{Z}_p[G/H]$ for some subgroup H of G.

The connection to representation theory cannot be used in a straightforward way, however. Indeed, if one factors out the commutator subgroup of a free open normal subgroup F then the obtained G/F-module would, in general, not satisfy the hypothesis of Weiss' theorem. In order to make representation theory work, we use pro-p HNN-extensions to embed G into a rather special virtually free pro-p group $\tilde{G}$, in which, after factoring out the commutator of a free open normal subgroup, the hypotheses of Weiss' theorem are satisfied. With its aid we prove Theorem 1 for $\tilde{G}$ and apply the Kurosh subgroup theorem to deduce the result for G.

We use notation for profinite and pro-p groups from [6].

2 Preliminary results

Theorem 2.1 *((2.6) Theorem in [3]) Let G be a group of order p and M a $\mathbf{Z}_p[G]$-module, free as a $\mathbf{Z}_p$-module. Then*

$$M = M_1 \oplus M_p \oplus M_{p-1}$$

such that M_p is a free G-module, M_1 is a trivial G-module and on M_{p-1} the equality $1 + c + \cdots + c^{p-1} = 0$ holds, where $G = \langle c \rangle$.

Let G be a p-group. A *permutation lattice* for G means a direct sum of G-modules, each of the form $\mathbf{Z}_p[G/H]$ for some subgroup H of G. A permutation lattice will be also called *G-permutational* module.

If G is of order p then Theorem 2.1 implies that M is permutational lattice if and only if M_{p-1} is missing in the decomposition for M if and only if $M/(g-1)M$ is torsion free for $1 \neq g \in G$.

Lemma 2.2 *For any finite group N, integral domain R and finitely generated free $R[N]$-module M the map $\phi : M \to M^N$ defined by $\phi(m) := \sum_{n \in N} nm$ is an epimorphism with kernel JM, where J is the augmentation ideal in $R[N]$.*

Proof: The map is well-defined, since $\sum_{n \in N} n$ belongs to the centre of $R[N]$ and for the same reason JM is contained in the kernel of ϕ. Present $M = R[N] \otimes_{R[N]} L$ with L a free R-module, then when $m = \sum_{n \in N} n \otimes l(n)$ with $l(n) \in L$ belongs to M^N, it means that all $l(n)$ are equal, so that $m = \phi(1_N \otimes l(n))$. Hence ϕ is an epimorphism. If $m = \sum_{x \in N} x \otimes l(x) \in \ker(\phi)$, then mod JM it is of the form $\sum_{x \in N} 1_N \otimes l(x)$ and therefore $\sum_{x \in N} l(x) = 0$ must hold, i.e., $m \in JM$.

Remark 2.3 When applying Theorem 1.2, in light of Lemma 2.2, we usually shall check the hypothesis for M_N instead of M^N.

We shall need the following connection between free decompositions and $\mathbf{Z}_p$-representations for free pro-p by C_p groups.

Lemma 2.4 *Let G be a split extension of a free pro-p group F of finite rank by a group of order p. Then*

(i) ([8]) G has a free decomposition $G = \left(\coprod_{a\in A} C_a \times H_a\right) \amalg H$, with $C_a \cong C_p$ and H_a and H free pro-p.

(ii) Set $M := F/[F,F]$. Fix $a_0 \in A$ and a generator c of C_{a_0}. Then conjugation by c induces an action of C_{a_0} upon M. The latter module decomposes in the form

$$M = M_1 \oplus M_p \oplus M_{p-1}$$

such that M_p is a free $\langle c\rangle$-module, on M_{p-1} the equality $1 + c + \cdots + c^{p-1} = 0$ holds, and c acts trivially on M_1.

Moreover, the ranks of the three G/F-modules satisfy $\operatorname{rank}(M_p) = \operatorname{rank}(H)$, $\operatorname{rank}(M_{p-1}) = |A| - 1$, and, $\operatorname{rank}(M_1) = \sum_{a\in A} \operatorname{rank}(H_a)$.

In particular, M is G/F-permutational if and only if $|A| = 1$.

Proof: (i) is 1.1 Theorem in [8]. For proving (ii), first pick $a_0 \in A$, second, for each $a \in A$ a generator c_a of C_a, and put $c_{a_0} := c$. We claim that

$$F = \left(\coprod_{a\in A} H_a\right) \amalg \left(\coprod_{j=0}^{p-1} H^{c^j}\right) \amalg \left(\coprod_{a\in A\setminus\{a_0\}} \coprod_{j=0}^{p-1} \langle c_a c^{-1}\rangle^{c^j}\right).$$

Indeed, consider the epimorphism $\phi : G \to C_p$ with H and all H_a in the kernel and sending each generator c_a of C_a to the generator of C_p. Then clearly $F = \ker\phi$ equals $\langle H^{c^j}, H_a, (c_a c^{-1})^{c^j} \mid a \in A, j = 0, \ldots, p-1\rangle$. This shows that

$$\operatorname{rank}(F) \le \sum_{a\in A} \operatorname{rank} H_a + p \operatorname{rank}(H) + (p-1)(|A| - 1).$$

On the other hand, one can use the pro-p-version of the Kurosh subgroup theorem, Theorem 9.1.9 in [6] applied to F as an open subgroup of G, to see that

$$F = \left(\coprod_{a\in A} H_a\right) \amalg \left(\coprod_{j=0}^{p-1} H^{c^j}\right) \amalg U$$

with U a free pro-p subgroup of F having $\operatorname{rank}(U) = 1 + |A|p - |F\backslash G/H| - \sum_{a\in A} |F\backslash G/(H_a \times C_a)| = 1 + |A|p - p - |A| = (|A| - 1)(p - 1)$. It shows the validity of the claimed free decomposition of F.

Factoring out $[F,F]$ yields the desired decomposition – the images of the three free factors. Finally, M_{p-1} appears as follows: writing $f_a := c_a c^{-1}$ a straight forward calculation yields the equality $f_a^{c^{p-1}} f_a^{c^{p-2}} \cdots f_a^c f_a = 1$ for every $a \in A$, which, in additive notation, reads $(c^{p-1} + c^{p-2} + \cdots + c + 1)\bar{f}_a = 0$.

Corollary 2.5 *If for each $a \in A$ a basis B_a of H_a is given and B is any basis of H, then $\bigcup_{a\in A} B_a[F,F]/[F,F]$ is a basis of M_1 and $B[F,F]/[F,F]$ a basis of the G/F-module M_p. A basis of M_{p-1} is given by $\{c_a c_{a_0}^{-1} \mid a \in A, a \neq a_0\}[F,F]/[F,F]$.*

Lemma 2.6 *Every finitely generated virtually free pro-p group has, up to conjugation, only a finite number of finite subgroups.*

Proof: Suppose that the lemma is false and that G is a counter-example possessing a normal free pro-p subgroup F of minimal possible index. When H is a maximal open subgroup of G with $F \leq H$ then, as $|H:F| < |G:F|$, the proper subgroup H satisfies the conclusion of the lemma, and so there are, up to conjugation, only finitely many finite subgroups of G, contained in H. Hence, in order to be a counter-example, G must be of the form $G = F \rtimes K$ for a finite subgroup K of G and, as G contains only finitely many such subgroups H, the proof is finished, if we can show that up to conjugation, there are only finitely many finite subgroups $L \cong K$ in G. Let t be a central element of order p in K and consider $G_1 := F \rtimes \langle t\rangle$. Certainly G_1 is finitely generated. Hence, as a consequence of Lemma 2.4(i), G_1 satisfies the conclusion of the lemma, and so, $G > G_1$. Next observe that any finite subgroup $L \cong K$ of G containing some torsion element $s \in G_1$ is contained in $C_G(s)$. By 1.2 Theorem in [8], $C_F(s)$ is a free factor of F and therefore, since F is finitely generated, $C_F(s)$ is finitely generated as well, and so is $C_G(s)$. Let bar denote passing to the quotient mod the normal subgroup s of $C_G(s)$. Then $|\overline{C_G(s)} : \overline{C_F(s)}| < |G:F|$, so that $\overline{C_G(s)}$ contains only finitely many conjugacy classes of maximal finite subgroups. Since the centralizers of conjugate elements are conjugate, G can, up to conjugation, contain only finitely many maximal finite subgroups, a contradiction.

Lemma 2.7 *Let G be virtually free pro-p and $C_F(t) = \{1\}$ for every torsion element $t \in G$. Then any pair of distinct maximal finite subgroups A, B of G has trivial intersection.*

Proof: Suppose that the lemma were false. Then one can pick maximal finite subgroups A and $B \neq A$ such that $1 \neq C := A \cap B$ is of maximal possible cardinality. Then C is a finite normal subgroup of $L := \langle N_A(C), N_B(C)\rangle$, so the latter is itself finite, since $N_G(C)$ must be finite (by Lemma 9.2.8 in [6] a finite normal subgroup of a pro-p group intersects the centre non-trivially). On the other hand, one must have $L \cap A = C$ due to the maximality assumption on the cardinality of pairwise intersections of maximal finite subgroups. Since $C < A$ one arrives at the contradiction $C < N_A(C) \leq L \cap C = C$.

We shall frequently use also the following results about virtually free pro-p groups and free pro-p products.

Proposition 2.8 *[([10], Proposition 1.7)] Let G be a virtually free pro-p group. Then*

(i) $G/\langle \mathrm{Tor}(G)\rangle$ *is free pro-p;*

(ii) $\mathrm{Tor}(G)$ *maps onto* $\mathrm{Tor}(G/\langle \mathrm{Tor}(G)\rangle)$ *under the canonical epimorphism* $G \longrightarrow G/\langle \mathrm{Tor}(G)\rangle$.

Theorem 2.9 *[([6], Theorems 9.1.12 and 9.5.1)] Let $G = \coprod_{i=1}^{n} G_i$ be a free profinite (pro-p) product. Then $G_i \cap G_j^g = 1$ for either $i \neq j$ or $g \notin G_j$.*

Every finite subgroup of G is conjugate to a subgroup of a free factor.

3 HNN-embedding

We introduce a notion of a pro-p HNN-group as a generalization of pro-p HNN-extension in the sense of [7], page 97. It also can be defined as a sequence of pro-p HNN-extensions. During the definition to follow, i belongs to a finite set I of indices.

Definition 3.1 Let G be a pro-p group and A_i, B_i be subgroups of G with isomorphisms $\phi_i : A_i \longrightarrow B_i$. The pro-$p$ HNN-group is then a pro-p group $HNN(G, A_i, \phi_i, z_i)$ having presentation

$$HNN(G, A_i, \phi_i, z_i) = \langle G, z_i \mid rel(G), \forall a_i \in A_i : \quad a_i^{z_i} = \phi_i(a_i)\rangle.$$

The group G is called the *base group*, A_i, B_i are called *associated subgroups* and z_i are called the *stable letters*.

For the rest of this section let G be a finitely generated virtually free pro-p group, and fix an open free pro-p normal subgroup F of G of minimal index. Also suppose that $C_F(t) = \{1\}$ for every torsion element $t \in G$. Let $K := G/F$ and form $G_0 := G \amalg K$. Let $\psi : G \to K$ denote the canonical projection. It extends to an epimorphism $\psi_0 : G_0 \to K$, by sending $g \in G$ to $gF/F \in K$ and each $k \in K$ identically to k, and using the universal property of the free pro-p product. Remark that the kernel of ψ_0, say L, is an open subgroup of G_0 and, as $L \cap G = F$ and $L \cap K = \{1\}$, as a consequence of the pro-p version of the Kurosh subgroup theorem, Theorem 9.1.9 in [6], L is free pro-p. Let I be the set of all G-conjugacy classes of maximal finite subgroups of G. Fix, for every $i \in I$, a finite subgroup K_i of G in the G-conjugacy class i. We define a pro-p HNN-group by considering first $\tilde{G}_0 := G_0 \amalg F(z_i \mid i \in I)$ with z_i constituting a free set of generators, and then taking the normal subgroup R in $\tilde{G}_0$ generated by all elements of the form $k_i^{z_i}\psi(k_i)^{-1}$, with $k_i \in K_i$ and $i \in I$. Finally set

$$\tilde{G} := \tilde{G}_0/R,$$

and note that it is an HNN-group $HNN(G_0, K_i, \phi_i, z_i)$, where $\phi_i := \psi_{|K_i}$, G_0 is the base group, the K_i are associated subgroups, and the z_i form a set of stable letters in the sense of Definition 3.1.

Let us show that $\tilde{G}$ is virtually free pro-p. The above epimorphism $\psi_0 : G_0 \longrightarrow K$ extends to $\tilde{G} \longrightarrow K$ by the universal property of the HNN-extension, so $\tilde{G}$ is a semidirect product $\tilde{F} \rtimes K$ of its kernel $\tilde{F}$ with K. By Lemma 10 in [5], every open torsion free subgroup of $\tilde{G}$ must be free pro-p, so $\tilde{F}$ is free pro-p.

The objective of the section is to show that the centralizers of torsion elements in $\tilde{G}$ are finite.

Lemma 3.2 *Let $\tilde{G} = HNN(G_0, K_i, \phi_i, z_i)$ and $\tilde{F}$ be as explained. Then $C_{\tilde{F}}(t) = 1$ for every torsion element $t \in \tilde{G}$.*

Proof: There is a standard pro-p tree $S := S(\tilde{G})$ associated to $\tilde{G} := HNN(G_0, K_i, \phi_i, z_i)$ on which $\tilde{G}$ acts naturally such that the vertex stabilizers are conjugates of G_0 and each edge stabilizer is a conjugate of some K_i (cf. [7] and §3 in [11]).

Claim: Let e_1, e_2 be two edges of S with a common vertex v. Then the intersection of the stabilizers $\tilde{G}_{e_1} \cap \tilde{G}_{e_2}$ is trivial.

By translating e_1, e_2, v if necessary we may assume that G_0 is the stabilizer of v. Then, up to orientation, we have two cases:

1) v is initial vertex of e_1 and e_2. Then $\tilde{G}_{e_1} = K_i^g$ and $\tilde{G}_{e_2} = K_j^{g'}$ with $g, g' \in G_0$ and either $i \neq j$ or $g \notin K_i g'$. Suppose that $K_i^g \cap K_j^{g'} \neq \{1\}$. Then, since $G_0 = G \amalg K$, we may apply Theorem 2.9, in order to deduce the existence of $g_0 \in G_0$ with $K_i^{gg_0} \cap K_j^{g'g_0} \leq G$. Now apply Lemma 2.7, in order to deduce the contradiction $i = j$ and $gg_0 \in K_i g' g_0$. So we have $K_i^g \cap K_j^{g'} = \{1\}$, as needed.

2) v is the terminal vertex of e_1 and the initial vertex of e_2. Then $\tilde{G}_{e_1} = K^g$ and $\tilde{G}_{e_2} = K_i^{g'}$ for $g, g' \in G_0$ so they intersect trivially by the definition of G_0 and Theorem 2.9. So the Claim holds.

Now pick a torsion element $t \in \tilde{G}$ and $f \in \tilde{F}$ with $t^f = t$. Let $e \in E(S)$ be an edge stabilized by t. Then fe is also stabilized by t and, as by Theorem 3.7 in [7], the fixed set S^t is a subtree, the path $[e, fe]$ is fixed by t as well. By the above then $fe = e$ contradicting the freeness of the action of $\tilde{F}$ on $E(S)$.

4 Proof of the main result

Proposition 4.1 *Let G be a semidirect product of a free pro-p group F of finite rank with a p-group K such that every finite subgroup is conjugate to a subgroup of K. Suppose that $C_F(t) = \{1\}$ holds for every torsion element $t \in G$. Then $G = K \amalg F_0$ for a free pro-p factor F_0.*

Proof: Suppose that the proposition is false. Then there is a counter-example with K having minimal order. When $K \cong C_p$, then by Lemma 2.4(i) $G = \left(\coprod_{i \in I} C_i\right) \amalg H$ with I a finite set, all C_i of order p and H free pro-p. By

the assumptions and Theorem 2.9 there is a single conjugacy class of finite subgroups, i.e., $|I| = 1$, so that G would not be a counter-example. Therefore K is of order $\geq p^2$.

Let H be any maximal subgroup of K. Then $F \rtimes H$ satisfies the premises of the proposition and hence $F \rtimes H$ is of the form $H \amalg F_1$ for some free factor F_1. Let us denote by bar passing to the quotient mod $(H)_G$. As $(H)_G = \langle \mathrm{Tor}(FH) \rangle$ by Proposition 2.8(i) $\bar{F}$ is free pro-p. Lemma 2.4(i) shows that $\bar{G} \cong \coprod_{i \in I} (C_i \times C_{\bar{F}}(C_i)) \amalg F_0$ with I finite and F_0 a free factor of $\bar{F}$. Now by Proposition 2.8(ii) $\overline{\mathrm{Tor}(\bar{G})} = \mathrm{Tor}(\bar{G})$, and therefore, every torsion element in $\bar{G}$ can be lifted to a conjugate of an element in K. Hence I consists of a single element, so that

$$\bar{G} = (\bar{K} \times C_{\bar{F}}(\bar{K})) \amalg F_0. \tag{1}$$

In the sequel we shall use Lemma 2.4(ii) a couple of times. Consider $M := F/F'$ as a K-module and let J denote the augmentation ideal of $\mathbf{Z}_p[H]$. Since $F \rtimes H = H \amalg F_1 = \left(\coprod_{h \in H} F_1^h\right) \rtimes H$, H acts by permuting the free factors F_1^h, so that M is a free H-module. Passing in Eq.(1) to the quotient mod the commutator subgroup of $\bar{F} = (C_{\bar{F}}(\bar{K}), F_0)_{\bar{G}}$, using Lemma 2.4, one can see that M/JM is a $\bar{K}$-permutation lattice. Then an application of Theorem 1.2 together with Remark 2.3 shows that M itself is a K-permutation lattice.

We shall show that M is a free K-module. Indeed, if any of the summands is not free, a proper subgroup of K, say S, acts trivially there. Since M is a free H-module, conclude that $S \cap H = \{1\}$, and from this that S is of order p. Let us show that $G_1 := F \rtimes S$ satisfies the premises of the proposition. Certainly $C_F(t) = \{1\}$ for every torsion element $t \in G_1$. Pick $x \in \mathrm{Tor}(G_1)$. There is $k \in K$ and $f \in F$ with $x = k^f$. Since $k \in (FS) \cap K$ deduce $k \in S$. So there is a single conjugacy class of finite subgroups in G_1. But then, considering the natural homomorphism from $F \rtimes S$ to $M \rtimes S$ and, observing the minimality assumption on $|K| > |S| = p$, so that $F \rtimes S = S \amalg F_S$ for some free pro-p group F_S, one finds as an application of Lemma 2.4 that the decomposition of M cannot have direct summands, on which S acts trivially, a contradiction. Since M is a K-permutational lattice, it is S-permutational as well and so cannot contain $p-1$ blocks, so that M is a free K-module.

Consider $\tilde{G} := K \amalg \tilde{F}_0$ with $\tilde{F}_0 \cong G/\langle \mathrm{Tor}(G) \rangle$. By Proposition 2.8 $G/\langle \mathrm{Tor}(G) \rangle$ is free pro-p, so we can fix a section F_0 of $G/\langle \mathrm{Tor}(G) \rangle$ inside G, and define a homomorphism $\phi : \tilde{G} \to G$ by first sending K to K and $\tilde{F}_0$ onto F_0 and then, using the universal property of the free product, extending it to $\tilde{G}$. By assumption all torsion elements are, up to conjugation, contained in K, showing that ϕ is an epimorphism. By the above the kernel of ϕ must be contained in $[\tilde{F}, \tilde{F}]$. In particular, since the group is finitely generated, one has $\tilde{F} \cong F$, since both groups are free pro-p. Since $K \cap \ker \phi = \{1\}$, conclude that ϕ is an isomorphism, as claimed.

Proof: [of Theorem 1.1:] Lemma 2.6 shows that G can have only a finite number of conjugacy classes of maximal finite subgroups. Therefore one can form $\tilde{G}$ as described before Lemma 3.2, in order to embed G such that $\tilde{G}$ is both, finitely generated, and, has finite centralizers of its finite subgroups, and, moreover, has

a single conjugacy class of maximal finite subgroups. By Proposition 4.1 the group $\tilde{G}$ is of the form $\tilde{G} = K \amalg F_0$ where K is finite and F_0 is free pro-p. Since G is a finitely generated pro-p subgroup of $\tilde{G}$, the Kurosh subgroup theorem in [4] implies that G must have indeed the form as claimed.

5 An example

We give an example of a virtually free profinite group that satisfies the centralizer condition of the main theorem but does not satisfy its conclusion. Note that the same example is valid for abstract groups.

Lemma 5.1 *Let $A \cong B = S_3$ be the symmetric group on a 3-element set and $C := C_2$. Form the amalgamated free profinite product $G = A \amalg_C B$, where C identifies with given 2-Sylow subgroups in A and B respectively.*

Then for every torsion element $t \in G$ its centralizer is finite. However, G cannot be decomposed as a free profinite product with some factor finite.

Proof: It is easy to see that G can be presented in the form $G = N \rtimes C_2$, with $N \cong C_3 \amalg C_3$ and $C_2 = \langle \alpha \rangle$ acting by inverting the generators of the two factors. Then the structure of N, in light of Theorem 2.9, shows that no element of order 3 can have an infinite centralizer. For establishing the first statement of the Lemma, it will suffice to show that all involutions in G are conjugate, and that α acts without fixed points upon $N = \langle a, b \rangle$, where a, b are generators of cyclic free factors of order 3. As G is the fundamental group of the graph of groups $\overset{A}{\bullet}\xrightarrow{\;C\;}\overset{B}{\bullet}$, Theorem 5.6 on page 938 in [11] shows that every involution is conjugate to an involution in one of the vertex groups A or B. As A and B both have a single conjugacy class of involutions and the latter contains C, the first observation holds. Since, by Theorem 9.1.6 in [6], N' is freely generated by the commutators $[a^i, b^j]$ with $i, j \in \{1, 2\}$, one can see that α permutes them without fixed points, so that $N' \rtimes \langle \alpha \rangle$ is isomorphic to $F(x, y) \amalg C_2$ with $F(x, y)$ a free profinite group. Thus α has no fixed points in N' and, as an easy consequence, none in N.

Suppose that $G = L \amalg K$ with L finite. Then, by Theorem 2.9, w.l.o.g. we can assume that $A \leq L$. The just cited Theorem on page 938 in [11] shows that A is a maximal finite subgroup of G, so that $A = L$. Since the quotient mod the normal closure of L in G is isomorphic to K on the one hand and trivial by construction, find $K = \{1\}$, a contradiction. So G has no finite free factor.

A list of remarks of a referee of a previous version of the paper led to immense improvement in presenting some proofs.

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